The Optimal Fuzzy Weight Set for Building a Model of Fuzzy Integrated Judgment

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Abstract

In the textile industry, cloth is made from many different brands of yarns, while yarns are made from various different kinds of cotton. Naturally, we want to achieve a full-fledged utilization of combining different brands of cotton to produce yarns with the best quality. Such a combination of different brands of cotton in the manufacturing process is called cotton allocation. In this regard, it often encounters a problem of how to select a substitute (from the warehouse) for a brand of cotton in the cotton allocation that is currently in use but running out of the raw material. A model using Fuzzy Dynamic Integrated Judgment (FDIJ) provides a very useful tool in resolving this problem. In view of several factors in cotton allocations, the FDIJ is made for cotton selection according to the data feedback from previous production runs. In [4], we explained how the data feedback can be stored in a fuzzy dynamic relationship dictionary which generates a fuzzy dynamic relation to be used in the learning ability in the problem of cotton allocation. It should be noted that finding a fuzzy weight set in the FDIJ is an inverse operation of fuzzy integrated judgment. In applications, it requires an initial fuzzy weight set (or the so-call the original image) to make the whole process work. The main objective of this paper is to explore a practical approach of obtaining the optimal fuzzy weight set for building a model of FDIJ whether or not the exact original image exists.

Keywords

Fuzzy Integrated Judgment; Fuzzy Weight Set; Fuzzy Feedback Relation; Similar Distance; Similar Degree; Fuzzy Similarity Set; Optimal Fuzzy Weight Set

Introduction

Professor L. A. Zadeh formalized Fuzzy Set Theory at the University of California at Berkeley in1965. The development of the theory of fuzzy sets was motivated largely by the need for computational framework for dealing with systems in which human judgment, behavior and emotions play a dominant role. The theory of fuzzy sets provides a much better model for human cognition than traditional approaches. One of the applications in the fuzzy set theory is the fuzzy integrated judgment. Areas in which the fuzzy integrated judgment has been successfully applied are often quite tangible. Finding a fuzzy weight set is an inverse operation of the fuzzy integrated judgment. However, the fuzzy weight set $\tilde{A}$, which stands for the experience or the technology of experts, is so important in determining whether or not the fuzzy integrated judgment is successful that an inverse problem of the fuzzy integrated judgment is not only important but also very practical.

As demonstrated in Zhao, Peng, and Cheng [6], the fuzzy dynamic integrated judgment (FDIJ) $\tilde{B}$ can be decided through the fuzzy relation equation $\tilde{B} = \tilde{A} \circ \tilde{R}$ based on an appropriate fuzzy weight set $\tilde{A}$ and an adjustable fuzzy relation (feedbacks) $\tilde{R}$. An application of FDIJ can be found in Zhao, Peng, and Cheng [6]. In general, finding $\tilde{A}$ when $\tilde{R}$ and $\tilde{B}$ are given is much more difficult than finding $\tilde{B}$ when $\tilde{R}$ and $\tilde{A}$ are given. In theory, for a given pair of $\tilde{B}$ and $\tilde{R}$, there may not be a feasible fuzzy weight set $\tilde{A}$ to satisfy the fuzzy relation equation $\tilde{B} = \tilde{A} \circ \tilde{R}$. In many applications, however, it is often encountered with the situation in which an optimal fuzzy weight set $\tilde{A}$ is sought to work with an adjustable fuzzy feedback relation $\tilde{R}$ in order to induce a desired fuzzy integrated judgment $\tilde{B}$. A feasible approach of deciding such an optimal fuzzy weight set $\tilde{A}$ for the desired fuzzy integrated judgment $\tilde{B}$ is presented in Section 4.

Some Basic Definitions

A crisp (classical) set is normally defined as a collection of elements or objects which can be finite or infinite. Each single element can either belong to or not belong to a set A, $A \subseteq X$. In the former case, the statement "x belongs to A" is true, whereas in the latter case this statement is false. There are many different approaches to describe a crisp set. One can either

enumerate (or list) the elements that belong to the set and describe the set analytically by stating conditions for membership such as $A = \{x \mid x \leq 5\}$, or define the member elements by using the characteristic function, in which "1" indicates membership and "0" non-membership.

Let X be a collection of objects denoted generically by x. Then a fuzzy set $\tilde{A}$ on X is defined to be a set of ordered pairs given by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$, where X is the universe of discourse which consists of all possible values for an input to $\tilde{A}$, x denotes the element in X, and $\mu_{\tilde{A}}(x)$ is called the *membership function* or *grade of membership* of x in $\tilde{A}$, which maps X to a membership space M. The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. Often it makes more sense that the values of $\mu_{\tilde{A}}(x)$ are indicated by a value on the unit interval, that is, $M = [0, 1]$, with 0 representing "Absolute False" and 1 representing "Absolute Truth". Thus $\mu_{\tilde{A}}(x)$ specifies the degree of belief that x belongs to the fuzzy set $\tilde{A}$. The elements with a zero degree of membership function are normally not listed.

In fact, the notion central to fuzzy sets is the membership function $\mu_{\tilde{A}}(x)$. Once the universe of discourse X is fixed, it is the membership function $\mu_{\tilde{A}}(x)$ that defines the fuzzy set $\tilde{A}$ completely.

Let X and $Y \subseteq \boldsymbol{R}$ (the set of real numbers) be two universe of discourses, then

$$\tilde{R} = \{((x, y), \mu_{\tilde{R}}(x, y)) \mid (x, y) \in X \times Y\}$$

is called a fuzzy relation on $X \times Y$, where $\mu_{\tilde{R}}(x, y)$ is a membership of the fuzzy relation which measures the degree of belief that the pair (x, y) belongs to the fuzzy relation $\tilde{R}$.

One of the compositions of fuzzy relation, that has become the best known and the most frequently used one, is the max-min composition. The max-min composition is defined as follows: Let

$$\tilde{R}_1 = \{((x, y), \mu_{\tilde{R}_1}(x, y)) \mid (x, y) \in X \times Y\}$$

and

$$\tilde{R}_2 = \{((y, z), \mu_{\tilde{R}_2}(y, z)) \mid (y, z) \in Y \times Z\}$$

be two fuzzy relations. The *max-min* composition of $\tilde{R}_1$ and $\tilde{R}_2$ is the fuzzy set

$$\tilde{R}_1 \circ \tilde{R}_2 = \{((x, z), max\{min\{\mu_{\tilde{R}_1}(x, y), \mu_{\tilde{R}_2}(y, z)\}\}) \mid (x, y) \in \tilde{R}_1 \text{ and } (y, z) \in \tilde{R}_2\}$$

Throughout this article, the judgment is a result of evaluating an object. The *focal-factors* are defined to be some attributes (in the object to be judged) which affect the judgment on the object deeply. A judgment is called a *single judgment* if there is only one focal-factor in an object, and an *integrated judgment* if there is more than one focal-factor in an object. A judgment is said to be a *crisp judgment* if it depends on a precise information source and a *fuzzy judgment* if it depends on imprecise or vague information source. We assign to each focal-factor a weight to indicate the impact of the focal-factor on the judgment (on an object).

We shall begin our discussion of the fuzzy dynamic integrated judgment (FDIJ) with the crisp integrated judgment. Focal-factor set is a crisp set which includes all focal-factors in an object to be judged, and is denoted as $U = \{u_i \mid i = 1, 2, \cdots, n\}$, where n is the number of focal-factors. Judgment set is a crisp set which includes all judgments we need in judging an object, and is written as $V = \{v_i \mid i = 1, 2, \cdots, m\}$, where m is the number of judgments. Fuzzy weight set $\tilde{A}$ is a fuzzy set which consists of all of weights corresponding to their focal-factors, and is denoted

$$\tilde{A} = \sum_{i=1}^{n} \frac{a_i}{u_i} \quad \text{or} \quad \tilde{A} = \{a_i \mid i = 1, 2, \cdots, n\}$$

where a_i is the weight corresponding to the i^{th} focal-factor, $u_i \in U$, n is the number of the focal-factor in U, and

$$\sum_{i=1}^{n} a_i = 1$$

Fuzzy single judgment set $\tilde{B}$ is a fuzzy set which consists of all of grades of membership corresponding to their judgments,

$$\tilde{B} = \sum_{j=1}^{m} \frac{\mu_{\tilde{B}}(v_j)}{v_j} \quad \text{or} \quad \tilde{B} = \{\mu_{\tilde{B}}(v_i) \mid j = 1, 2, \cdots, m\}$$

where $v_i \in V$, m is the number of judgments in V, and $\mu_{\tilde{B}}(v_j)$ is a grade of membership corresponding to the j^{th} judgment in V. Fuzzy integrated judgment $\tilde{B}$ can also be obtained by applying the *max-min* composition to a fuzzy weight set $\tilde{A}$ and a fuzzy relation set $\tilde{R}$, that is, $\tilde{B} = \tilde{A} \circ \tilde{R}$, and it can be enumerated as $\tilde{B} = \{b_j \mid j = 1, 2, \cdots, m\}$, where $b_j = \mu_{\tilde{B}}(v_j)$, and m is the number of judgments in V.

Fuzzy Similarity Set

Let A denote a collection of fuzzy sets on a universal set $X = \{x_1, x_2, \cdots, x_m\}$ with its members denoted generically by $\tilde{A}$. Let $\mu_{\tilde{A}}(x_i)$ denote a membership function of x_j in $\tilde{A}$.

Definition 1. Let $\tilde{A}$ and $\tilde{A}'$ be two fuzzy sets of A. The *similar distance* between $\tilde{A}$ and $\tilde{A}'$ is defined by

$$D(\tilde{A}, \tilde{A}') = \sqrt{\sum_{j=1}^{m}(\mu_{\tilde{A}}(x_j) - \mu_{\tilde{A}'}(x_j))^2}$$

Definition 2. The *fuzzy similarity set* is an ordered set of pairs given by

$$\tilde{S} = \{(\tilde{A}, \mu_{\tilde{S}}(\tilde{A})) \mid \tilde{A} \in A\}$$

where $\mu_{\tilde{S}}(\tilde{A})$ is a membership function of $\tilde{A}$ in $\tilde{S}$ defined by

$$\mu_{\tilde{S}}(\tilde{A}) = \frac{1}{1 + D(\tilde{A}, \tilde{A}')}$$

which is referred to as a *similar degree* between $\tilde{A}$ and $\tilde{A}'$.

Now we shall introduce an approach of creating the fuzzy similarity $\tilde{S}$ in order to find an optimal fuzzy weight set $\tilde{A}$ in building a model of FDIJ.

Step 1: Assume several possible fuzzy weight sets

$$\tilde{A}_k = \sum_{i=1}^{n}\frac{a_{ki}}{(focal-factor)_i} \quad \text{or} \quad \tilde{A}_k = (a_{k1}, a_{k2}, \cdots, a_{kn})$$

where a_{ki} is a weight of the i^{th} focal-factor in the fuzzy weight set $\tilde{A}_k$, k ranges from 1 to q (the number of fuzzy weight sets assumed) and i from 1 to n (the number of the focal-factors). Explicitly, we write

$$\tilde{A}_1 = (a_{11}, a_{12}, \cdots, a_{1n})$$

$$\tilde{A}_2 = (a_{21}, a_{22}, \cdots, a_{2n})$$

....

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....

$$\tilde{A}_q = (a_{q1}, a_{q2}, \cdots, a_{qn})$$

Step 2: Obtain a desired fuzzy integrated judgment,

$$\tilde{B} = \sum_{j=1}^{m}\frac{b_j}{(judgment)_j} \quad \text{or} \quad \tilde{B} = (b_1, b_2, \cdots, b_m)$$

where b_j is a value of the membership function corresponding to the j^{th} judgment and j ranges from 1 to m (the number of judgments).

Step 3: Obtain a fuzzy relation,

$$\tilde{R} = \{r_{ij} \mid i = 1, 2, \cdots, n \text{ and } j = 1, 2, \cdots, m\}$$

or

$$\tilde{R} = \begin{bmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & r_{nm} \end{bmatrix}$$

Step 4: By applying the *max-min* composition to $\tilde{A}_k$ and $\tilde{R}$, we calculate $\tilde{B}_k$ as follows:

$$\tilde{B}_k = \tilde{A}_k \circ \tilde{R} = \{a_{ki}\} \circ \{r_{ij}\}$$

with the result

$$\tilde{B}_k = \sum_{j=1}^{m}\frac{x_{kj}}{(judgment)_j} \quad \text{or} \quad \tilde{B}_k = (x_{k1}, x_{k2}, \cdots, x_{km})$$

where k ranges from 1 to q (the number of fuzzy weight sets assumed) and j ranges from 1 to m (the number of judgments). To be more explicit, we may write

$$\tilde{B}_1 = \tilde{A}_1 \circ \tilde{R} = (x_{11}, x_{12}, \cdots, x_{1m})$$

$$\tilde{B}_2 = \tilde{A}_2 \circ \tilde{R} = (x_{21}, x_{22}, \cdots, x_{2m})$$

....

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....

$$\tilde{B}_q = \tilde{A}_q \circ \tilde{R} = (x_{q1}, x_{q2}, \cdots, x_{qm})$$

Step 5: Build the fuzzy similarity $\tilde{S}$ as follows:

$$\tilde{S} = \{(\tilde{B}_k, \mu_{\tilde{S}}(\tilde{B}_k)) \mid \tilde{B}_k \in V\}$$

where $k \le q$, q is the number of fuzzy weight sets we assume, V is a judgment set, that is,

$$V = \{(judgment)_j \mid j = 1, 2, \cdots, m\}$$

The first term $\tilde{B}_k$ is a fuzzy integrated judgment corresponding to $\tilde{A}_k$ and the second term $\mu_{\tilde{S}}(\tilde{B}_k)$ is the similar degree of $\tilde{B}_k$ in $\tilde{S}$. In this case, the similar degree is given by

$$\mu_{\tilde{S}}(\tilde{B}_k) = \frac{1}{1 + D(\tilde{B}_k, \tilde{B})} \quad \text{for } k \le q$$

where $D(\tilde{B}_k, \tilde{B})$ is the similar distance between $\tilde{B}_k$ and $\tilde{B}$, that is,

$$D(\tilde{B}_k, \tilde{B}) = \sqrt{\sum_{j=1}^{m}(x_{kj} - b_j)^2} \quad \text{for } j \le m$$

Choosing an optimal Fuzzy Weight Set

It has been demonstrated that given a reasonable fuzzy weight set $\tilde{A}$ and an adjustable fuzzy feedback relation $\tilde{R}$, a powerful fuzzy integrated judgment $\tilde{B}$ can be obtained by repeatedly applying the fuzzy relation equation $\tilde{B} = \tilde{A} \circ \tilde{R}$. Now let us consider the inverse of this problem, that is, if a fuzzy integrated judgment $\tilde{B}$ is known and the fuzzy feedback relation $\tilde{R}$ is also available, then can we provide a legitimate fuzzy weight set, $\tilde{A}$? This problem is not as easy as it seems at a glance. Algebraically, solving the fuzzy equation $\tilde{B}$

= $\tilde{A} \circ \tilde{R}$ for $\tilde{A}$ may not be attainable. Therefore, an alternative approach may be necessary to take into account.

Let a fuzzy integrated judgment $\tilde{B}$ and a fuzzy feedback relation $\tilde{R}$ be given. Suppose that there are q fuzzy weight sets, $\tilde{A}_k$ (k = 1, 2, ⋯, q), that satisfy $\tilde{B}_k = \tilde{A}_k \circ \tilde{R}$ (k = 1, 2, ⋯, q), respectively, where$\tilde{B}_k$ (k = 1, 2, ⋯, q) are considered fuzzy similar to $\tilde{B}$. Then one of the fuzzy weight sets, $\tilde{A}_\alpha$, is said to be an *optimal fuzzy weight set* for $\tilde{B}$ if

$$\mu_{\tilde{S}}(\tilde{B}_\alpha) = max\{\mu_{\tilde{S}}(\tilde{B}_k) \mid k = 1, 2, \cdots, q\}$$

Example. In the problem of judging a specific fashion, we first choose three focal-factors,

$$U = \{style, color, price\}$$

and four judgments,

$$V = \{very\ good, good, not\ bad, bad\}$$

Suppose that a poll shows that 80 percent of customers consider the fashion good, 20 percent of customers consider it not bad, nobody consider it very good or bad. Then we can propose a desired fuzzy integrated judgment as follows:

$$\tilde{B} = \frac{0.0}{very\ good} + \frac{0.8}{very\ good} + \frac{0.2}{very\ good} + \frac{0.0}{very\ good}$$

or $\tilde{B}$ = (0.0, 0.8, 0.2, 0.0)

Further suppose that we have a given fuzzy relation given by

$$\tilde{R} = \begin{bmatrix} 0.2 & 0.2 & 0.1 & 0.0 \\ 0.0 & 0.4 & 0.5 & 0.1 \\ 0.2 & 0.3 & 0.4 & 0.1 \end{bmatrix}$$

Now we want to find an optimal fuzzy weight set for the desired fuzzy integrated judgment B. Assume that four fuzzy weight sets are chosen according to customers' psychology. They are

$$\tilde{A}_1 = \frac{0.2}{style} + \frac{0.4}{color} + \frac{0.3}{price} \quad \text{or} \quad \tilde{A}_1 = (0.2, 0.4, 0.3)$$

$$\tilde{A}_2 = \frac{0.5}{style} + \frac{0.4}{color} + \frac{0.2}{price} \quad \text{or} \quad \tilde{A}_2 = (0.5, 0.4, 0.2)$$

$$\tilde{A}_3 = \frac{0.2}{style} + \frac{0.3}{color} + \frac{0.5}{price} \quad \text{or} \quad \tilde{A}_3 = (0.2, 0.3, 0.5)$$

$$\tilde{A}_4 = \frac{0.7}{style} + \frac{0.26}{color} + \frac{0.4}{price} \quad \text{or} \quad \tilde{A}_4 = (0.7, 0.26, 0.4)$$

For each k = 1, 2, 3, 4, we now compute the corresponding fuzzy integrated judgment $\tilde{B}_k$ by applying the *max-min* composition to $\tilde{A}_k$ and $\tilde{R}$, respectively; that is,

$\tilde{B}_1 = \tilde{A}_1 \circ \tilde{R} = (x_{11}, x_{12}, \cdots, x_{1m})$ = (0.2, 0.4, 0.4, 0.1)

$\tilde{B}_2 = \tilde{A}_2 \circ \tilde{R} = (x_{21}, x_{22}, \cdots, x_{2m})$ = (0.2, 0.5, 0.3, 0.1)

$\tilde{B}_3 = \tilde{A}_3 \circ \tilde{R} = (x_{31}, x_{32}, \cdots, x_{3m})$ = (0.2, 0.3, 0.4, 0.1)

$\tilde{B}_4 = \tilde{A}_4 \circ \tilde{R} = (x_{41}, x_{42}, \cdots, x_{4m})$ = (0.2, 0.7, 0.26, 0.1)

Based on these four fuzzy integrated judgments $\tilde{B}_k$ (k = 1, 2, 3, 4), their similar distances $\mathrm{D}(\tilde{B}_k, \tilde{B})$ from $\tilde{B}$ are calculated, respectively, as follows:

$$\mathrm{D}(\tilde{B}_1, \tilde{B}) = \sqrt{\textstyle\sum_{j=1}^{m}(x_{1j} - b_j)^2} = 0.50$$

$$\mathrm{D}(\tilde{B}_2, \tilde{B}) = \sqrt{\textstyle\sum_{j=1}^{m}(x_{2j} - b_j)^2} = 0.39$$

$$\mathrm{D}(\tilde{B}_3, \tilde{B}) = \sqrt{\textstyle\sum_{j=1}^{m}(x_{3j} - b_j)^2} = 0.76$$

$$\mathrm{D}(\tilde{B}_4, \tilde{B}) = \sqrt{\textstyle\sum_{j=1}^{m}(x_{4j} - b_j)^2} = 0.25$$

and their similar degrees $\mu_{\tilde{S}}(\tilde{B}_k)$ to $\tilde{B}$ are also calculated, respectively, as follows:

$$\mu_{\tilde{S}}(\tilde{B}_1) = \frac{1}{1 + \mathrm{D}(\tilde{B}_1, \tilde{B})} = 0.6667$$

$$\mu_{\tilde{S}}(\tilde{B}_2) = \frac{1}{1 + \mathrm{D}(\tilde{B}_2, \tilde{B})} = 0.7194$$

$$\mu_{\tilde{S}}(\tilde{B}_3) = \frac{1}{1 + \mathrm{D}(\tilde{B}_3, \tilde{B})} = 0.5681$$

$$\mu_{\tilde{S}}(\tilde{B}_4) = \frac{1}{1 + \mathrm{D}(\tilde{B}_4, \tilde{B})} = 0.8000$$

Then the fussy similarity set is

S = {($\tilde{B}_1$, 0.6667), ($\tilde{B}_2$, 0.7194), ($\tilde{B}_3$, 0.5681), ($\tilde{B}_4$, 0.8000)}

Obviously, $\tilde{B}_4$ has the maximum similar degree of 0.8000, that is,

$$\mu_{\tilde{S}}(\tilde{B}_4) = max\{\mu_{\tilde{S}}(\tilde{B}_k) \mid \mathrm{k} = 1, 2, 3, 4\}$$

Therefore, the fuzzy set $\tilde{A}_4$, that induces $\tilde{B}_4$, is the optimal fuzzy weight set for the desired fuzzy integrated judgment $\tilde{B}$.

Conclusion and Future Direction

In building a model for the fuzzy integrated judgment (FIJ) or for the fuzzy dynamic integrated judgment (that is a kind of FIJ with a dynamic fuzzy relation for obtaining a learning ability and is simply written as FDIJ), it is often encountered with the problem of how to select an optimal fuzzy weight set $\tilde{A}$ for a desired

fuzzy integrated judgment $\tilde{B}$. We approach this problem with first introducing some new definitions such as fuzzy similarity set, similar degree, and similar distance, and then choosing a fuzzy weight set $\tilde{A}_\alpha$, whose corresponding $\tilde{B}_\alpha$ has the maximum value of similar degree to the desired fuzzy integrated judgment $\tilde{B}$, to be the optimal fuzzy weight set for $\tilde{B}$. We firmly belief that inverse problem is one of the center pieces in building an intelligent model of the fuzzy integrated judgment for expert system. Any research that attempts to integrate this inverse problem with computer applications is considered valuable.

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